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Random Vibration Theory-Based Soil-Structure Interaction Analysis

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SUMMARY
This paper presents an alternative approach of performing seismic soil structure interaction (SSI) analysis. In this approach, the input earthquake motion is represented by an acceleration response spectrum, instead of an acceleration time history fitted to the input spectra as currently used in most SSI analysis. The input spectrum is converted to a power spectrum density (PSD) function, and the response PSDs of the soil-structure system to the input PSD are computed. All required design parameters are computed from the PSD responses of the system. The solutions obtained as such represent the statistical mean from all possible input time histories fitting the same input spectrum.

Results of comparison studies indicate this new approach is compatible with all requirement of a typical SSI analysis, and the results show good agreements with the "conventional" SSI analysis results. A numerical example is included in this paper to illustrate the efficiency of the new approach

Keywords: Soil Structure Interaction Analysis, Random Vibration Theory, SASSI2010.

INTRODUCTION

Seismic soil-structure interaction (SSI) analysis is a critical part of the seismic safety evaluation for all nuclear reactors and other safety related structures and is required for seismic design. Currently, SSI analysis is mostly based on deterministic analysis with a set of acceleration time histories representing the design motion, such as the approach implemented in the computer program SASSI. The design motion is first defined through seismological study by a set of design acceleration response spectra (ARS), which can be either site-specific or site-independent. Synthetic time histories matching the design ARS are generated and are used as the input motion for SSI analysis. All quantities of structural responses, such as ARS, maximum accelerations, shear and axial forces in the structural components, etc., were computed as response time histories.

A major uncertainty in this approach is the appropriate selection of the seed time history for matching to the design spectrum and number of time histories needed to obtain stable mean SSI responses. Several recent studies have shown that use of different time histories as input motion, even with each of them closely matching to the target design ARS, may result in responses that can vary as much as 30% in maximum values and the in-structure response spectra. This observation has led the code committees and the regulatory agencies aiming for use of multiple time histories even for linear SSI analysis to ensure a stable mean response is captured. For example, the latest to-be-published revision of ASCE-4 (the update to ASCE-4, 1998) requires use of a minimum of five time histories for SSI analysis. Furthermore, at current time there are no specific guidelines for selection of seed time histories or the optimum number of time histories in order to obtain a stable mean response.

In this paper, an alternate approach is proposed to mitigate this uncertainty. This approach ensures a stable mean response while eliminating the efforts required in generating spectrum-matching multiple time histories and in post-processing of the SSI results from using these multiple time histories. In this approach the following main steps are taken:

- The input target rock response spectrum is first converted to a power spectrum density (PSD) function.
- The response PSDs of the soil-structure system with respect to the input motion are computed based on the input PSD and the transfer function solutions of the soil-structure system obtained through a standard SSI analysis.
- The output PSDs are converted back to the statistical means of the extreme values for all required quantities, such as acceleration responses spectra, maximum accelerations, maximum base shear, bend moments for beams, etc.

This approach is consistent with the RVT-based site response analysis approach developed and reported by the authors (Deng and Ostadan, 2008, 2011). The application of RVT is now extended to 2-D and 3-D SSI analysis and computation of structural responses. The formulation has been implemented in a new computer program SASSI2010 (Bechtel, 2011). Since the basic solution procedure of a standard SSI analysis is well known, this paper will present only the portion unique to the subject.

THEORY

Converting an Acceleration Response Spectrum to a Power Spectrum Density Function

It is well known from basic RVT theory (e.g., Der Kiureghian, 1983) that the following relation exists

$$S_d(\omega) = \left|H^2(\omega)\right| S_a(\omega) \tag{1}$$

where $S_d(\omega)$ is the relative displacement PSD, $S_a(\omega)$ is the acceleration PSD, and $H(\omega)$ is the transfer function between displacement response and absolute acceleration input of a single degree of freedom oscillator with frequency ω_o and damping ξ

$$\left|H^2(\omega)\right| = \frac{1}{(\omega_o^2 - \omega^2)^2 + 4\xi^2\omega_o^2\omega^2} \tag{2}$$

The mean of the maximum relative displacement response of the oscillator (definition of a mean relative displacement response spectrum) is given by:

$$D = p\sqrt{\lambda_0} \tag{3}$$

Where p is a peak factor, and λ_0 is the zero moment of the response defined in Equation (6). Following Davenport (1964) and Der Kiureghian (1980)

$$p = \sqrt{2\ln\nu(0)\tau} + \frac{0.5772}{\sqrt{2\ln\nu(0)\tau}} \tag{4}$$

$\nu(0)$ is the mean zero crossing of the response between 0 and τ and equal to:

$$\nu(0) = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{5}$$

where τ is taken as the strong motion duration of the earthquake.

The moments of the response are defined as the following

$$\lambda_n = \int_0^\infty \omega^n S_d(\omega) d\omega \tag{6}$$

$n = 0, 1, 2$ for the zero (λ_0), first (λ_1), and second (λ_2) moments of the response.

Following Igusa and Der Kiureghian (1983) and Venmarcke (1975), $\nu(0)$ necessarily is adjusted with the parameter δ, where

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \tag{7}$$

The steps to calculate the acceleration power spectral density function from a given acceleration response spectrum are as follows.

1. Convert the acceleration response spectrum $RS_a(\omega)$ to a relative displacement response spectrum $RS_d(\omega)$,
2. Assume an initial acceleration power spectral density function $S_{a,0}(\omega)$, usually a constant value of unity is assumed as the initial value over the frequency range.
3. With the assumed $S_{a,0}(\omega)$ and the relations given above, calculate the mean of the maximum relative displacement response for all the frequencies defining the response spectrum. This will be a new relative displacement response spectrum $RS_{d,1}(\omega)$.
4. Calculate the ratio $R(\omega) = RS_d(\omega)/RS_{d,1}(\omega)$.
5. Correct the assumed acceleration power spectral density function $S_{a,0}(\omega)$ by $R^2(\omega)$ to calculate a new acceleration power spectral density function $S_{a,1}(\omega)$
6. Iterate from step 3 to step 5 until the desired accuracy is reached in the calculation of the displacement response spectrum.

Determine the Mean of Maximum Responses

Transfer function solutions of any and all requested responses $H_r(\omega)$ are computed through a standard SSI analysis. $H_r(\omega)$ include solutions for the global equations and all required transforming operations to represent solutions for ARS, ZPA, stress and strain, forces and moments, etc. With the input motion PSD and the transfer function solutions, steps to calculate the mean of the maximum response are as following:

1. Calculate the PSD of the desired response $S_r(\omega)$

$$S_r(\omega) = \left| H_r^2(\omega) \right| S_a(\omega) \tag{8}$$

2. Calculate the moments $\lambda_{r,0}$, $\lambda_{r,1}$, $\lambda_{r,2}$ of the response

$$\lambda_{r,n} = \int_0^\infty \omega^n S_r(\omega) d\omega \tag{9}$$

3. Calculate the peak factor p_r with the same formulas as in Eqn. 4 thru 7, except all the values are now calculated with $\lambda_{r,n}$, $n = 0, 1, 2$

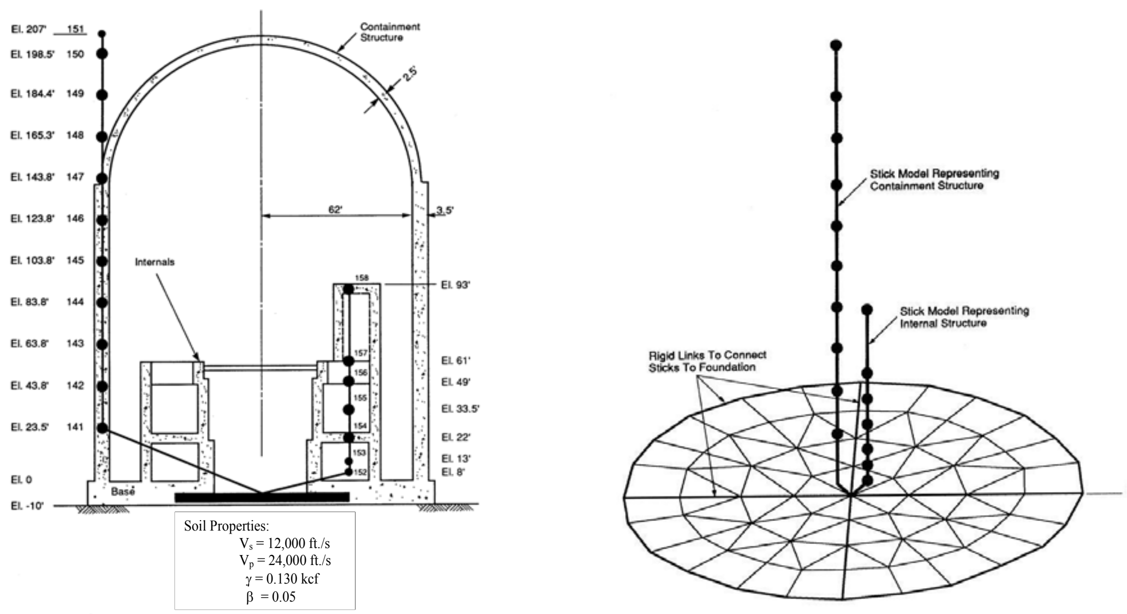
4. Calculate the mean of the maximum response, M_r

$$M_r = p_r \sqrt{\lambda_{r,0}} \tag{10}$$

NUMBERICAL EXAMPLE

A numerical example is shown below to illustrate the steps and efficiency of the new approach.

A nuclear containment building, as shown in Figure 1(a), is modelled by a simple lumped mass and stick model with rigid basemat, Figure 1(b). The model is at top of a rigid halfspace with the shear wave velocity V_s = 12,000 ft/sec, p-wave velocity V_p = 24,000 ft/s, unit weight γ = 0.130 kcf and fraction of critical damping β = 0.05. Details of the FEM model can be found in the SASSI user manual.



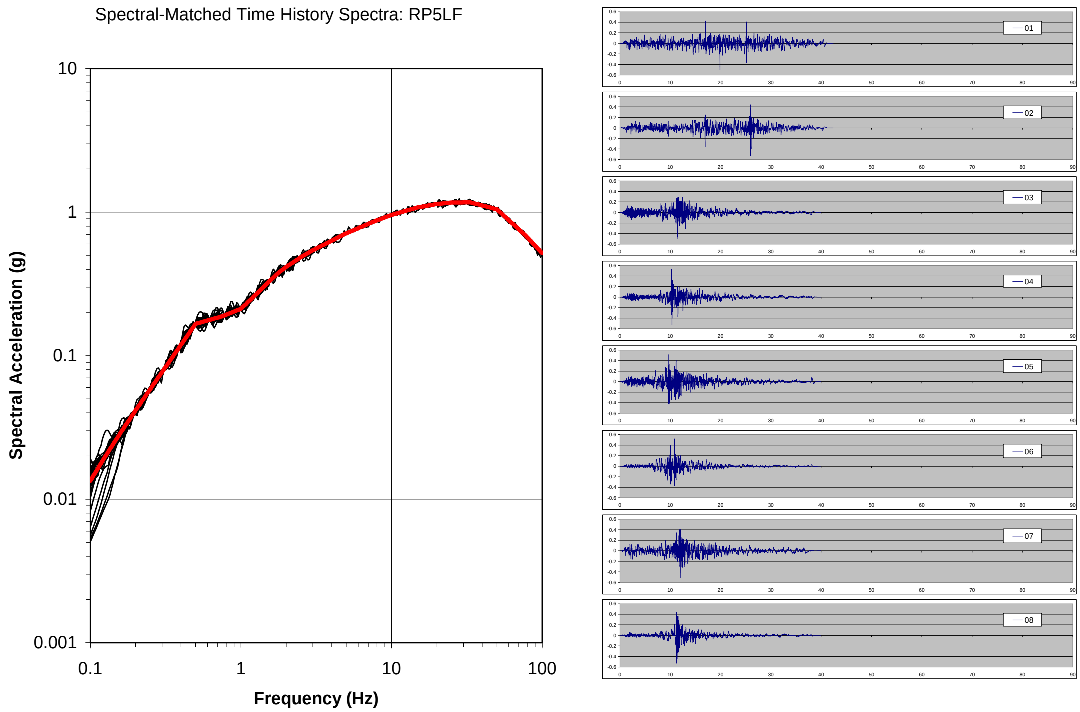
(a) **(b)**

Figure 1. (a) A Containment Building and (b) the Finite Element Model for SSI Analysis

A design response spectrum at the surface outcrop was developed through probabilistic seismic hazard analysis (Figure 2(a), the red line). Thirty (30) acceleration time histories were selected as the "seeds" from historical recordings around an Eastern US site and other earthquakes with similar geological and seismological conditions. These 30 time histories then modified to fit the design ARS. The matching is shown as thin black lines in Figure 2(a). Figure 2(b) shows a few of these matched time histories. The input motion was specified at the grade surface in horizontal direction.

Two parallel SSI analyses were performed to illustrate the efficiency of the new approach. First, a standard SSI analysis was performed 30 times, each time using a different time history, and 30 sets of analysis results were computed. The averages of these results were obtained. Next, one SSI analysis was performed and the results were computed using the RVT approach. The results in terms of averages of maximum acceleration, in-structure acceleration response spectra and maximum member forces are compared.

Figure 3 shows the comparison of the maximum acceleration (ZPA) at all lumped mass points. Figures 4, 5 and 6 show the comparison of 5% damped ARS at basemat, top of containment structure, and top of internal structures, respectively. Tables 1 and 2 show the comparisons of base shears and base moments for the containment and internal structures, respectively.



(a) **(b)**

Figure 2. Input Motion for the Analysis. (a) Design ARS and the overall matching for the time histories. (b) The first 8 time histories matched to the design spectrum.

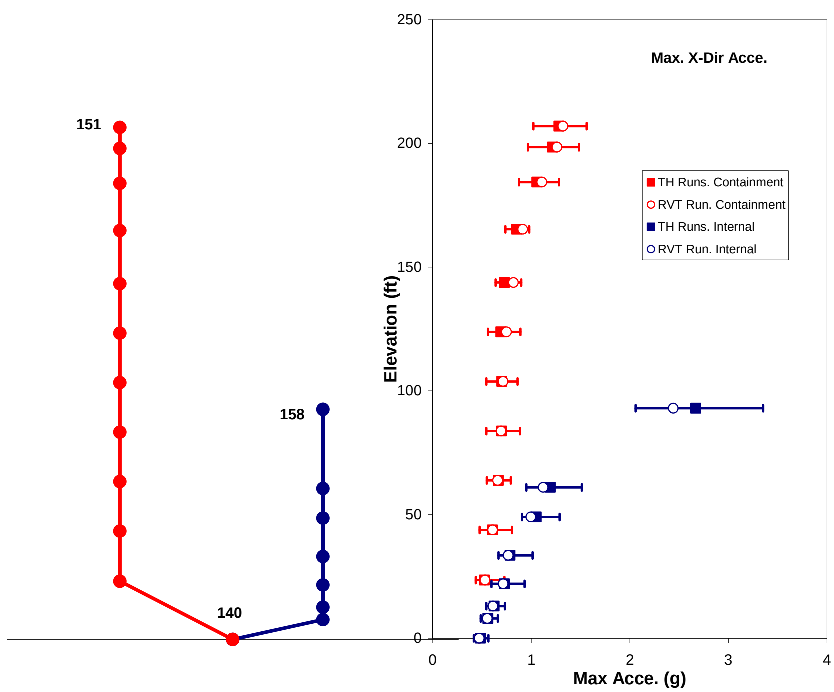


Figure 3. Comparison of calculated maximum accelerations (ZPAs) at all lumped mass points. In the figure, the sketch of the stick model and all lumped mass points are on the left and the computed ZPAs are on the right corresponding to the elevation of the mass points. The bars show the ranges of ZPA calculated from all time histories (TH). The solid squares are average ZPA for 30 TH runs. The hollow circles are results from RVT run.

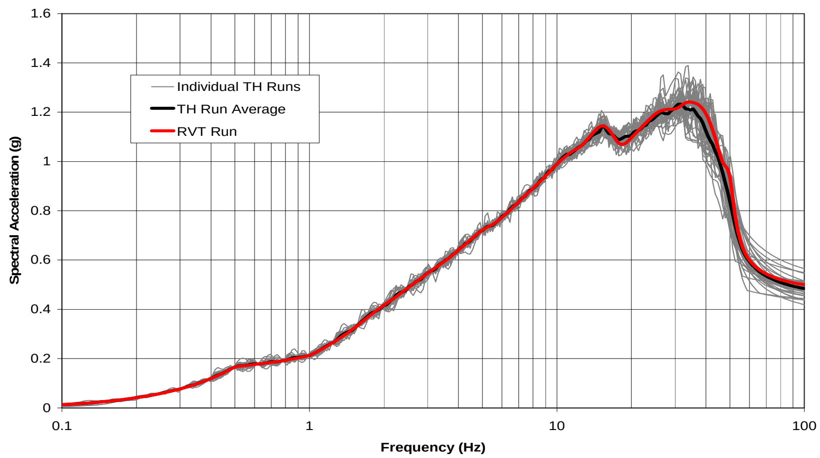


Figure 4. Comparison of 5% damped acceleration response spectra at the basemat. Thin grey lines are individual time history (TH) runs. Thick black line is the average of TH runs. Red line is the results from RVT run.

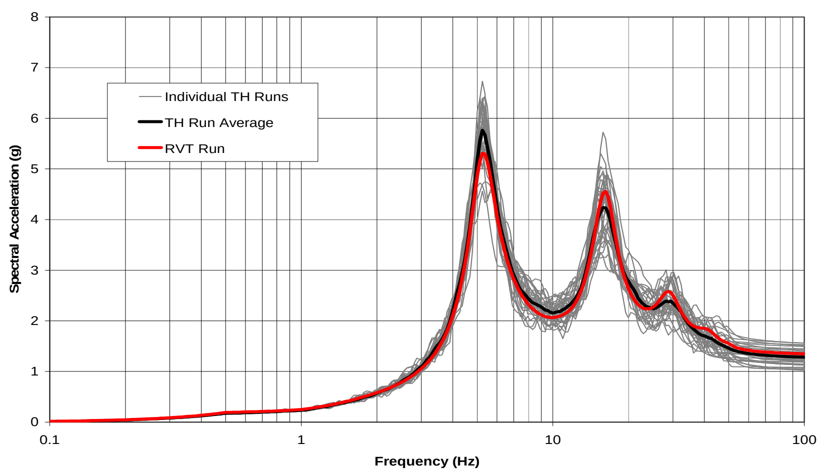


Figure 5. Comparison of 5% damped acceleration response spectra at top of containment structure.

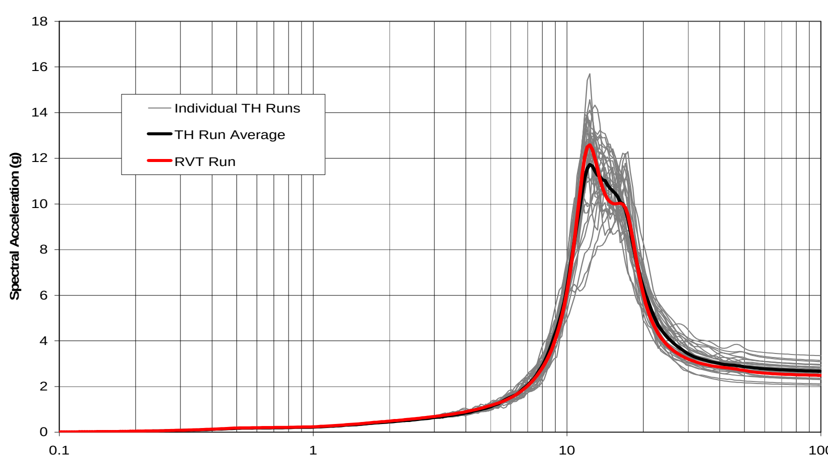


Figure 6. Comparison of 5% damped acceleration response spectra at top of internal structure

Table 1. Forces and Moments in the Containment Structure Stick

		Base Shear X-Dir (kips)	Base Moment YY-Dir (kip-ft)
30 TH Runs	Min	17790	2.283E+06
	Average	19764	2.446E+06
	Max	23130	2.670E+06
RVT Run		20980	2.449E+06
Differences		6.1 %	0.1 %

Table 2. Forces and Moments in the Internal Structure Stick

		Base Shear X-Dir (kips)	Base Moment YY-Dir (kip-ft)
30 TH Runs	Min	13870	4.866E+05
	Average	17350	6.438E+05
	Max	21130	7.771E+05
RVT Run		16690	6.196E+05
Differences		-3.8 %	-3.8 %

As shown in the results of the example presented above the RVT results are generally in good to excellent agreement with the average of TH results. However, standard TH-based SSI analysis requires 30 or more analyses while the RVT-based approach needs only one analysis to achieve essentially the same results.

The example presented in this paper is one of the many examples used for testing of the RVT approach in SASSI2010. Due to limitation of the space, those examples are not discussed in this paper.

CONCLUSIONS

An alternative approach for seismic soil structure interaction analysis is presented in this paper. This approach is based on the random vibration theory and is compatible with the theoretical framework of the computer program SASSI. In this approach, the design input motion is characterized by the design acceleration response spectrum directly, all intermediate computations are calculated through PSD and transfer functions, and all responses of interest are calculated as the statistical averages. This approach avoids the difficulties associated with generating multiple spectrum-matching input time histories and is very efficient in generating seismic design parameters from a SSI analysis. More importantly, this approach guards against any unconservatism that may arise from the use of single or limited time histories or use of inappropriate seed time histories by providing a stable mean response working directly with the design response spectra as input.

A numerical example is given in this paper to illustrate that the results computed by the new approach are in good agreement in terms of statistical average with the results computed time history based SSI analysis.

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